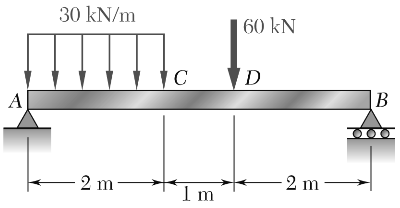
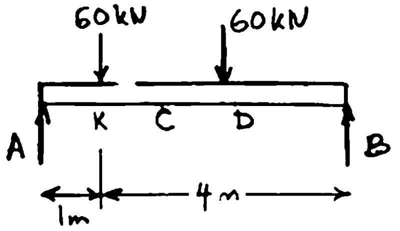


PROBLEM 5.10

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

Calculate reactions at *A* and *B*. Replace distributed load by an equivalent concentrated load after drawing the free body *ACDB*.



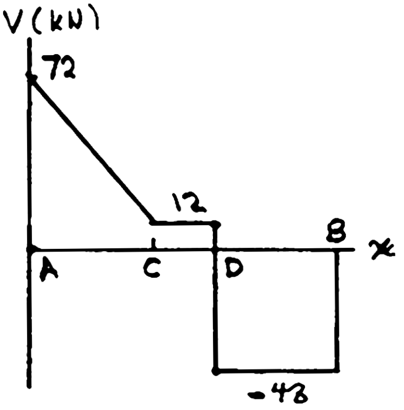
$$+\circlearrowleft \Sigma M_B = 0: \quad -5A + (4)(60) + (2)(60) = 0$$

$$A = 72\text{ kN}\uparrow$$

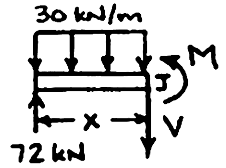
$$+\Sigma M_A = 0: \quad 5B - (1)(6) - (3)(60) = 0$$

$$B = 48\text{ kN}\uparrow$$

Check $+\uparrow \Sigma F_y = 0: \quad 72 - 60 - 60 + 48 = 0$



A to *C*. $0 < x \le 2\text{m}$



$+\uparrow \Sigma F_y = 0:$

$$72 - 30x - V = 0$$

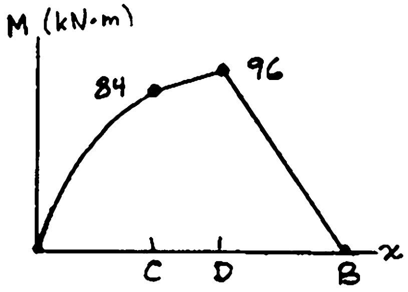
$$V = (72 - 30x)\text{ kN}$$

$+\circlearrowleft \Sigma M_J = 0:$

$$-72x + (30x)\frac{x}{2} + M = 0$$

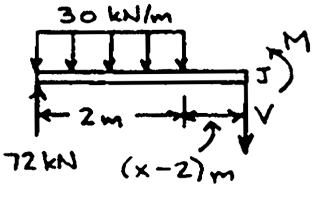
$$M = (72 - 15x^2)\text{ kN}\cdot\text{m}$$

C to *D*. $2\text{m} \le x < 3\text{m}$



$+\uparrow \Sigma F_y = 0: \quad 72 - (30)(2) - V = 0$

$$V = 12\text{ kN}$$

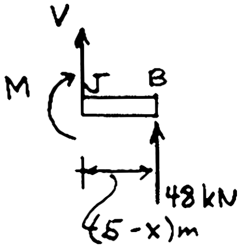


$+\circlearrowleft \Sigma M_J = 0:$

$$-72x + (30)(2)(x - 1) + M = 0$$

$$M = (12x + 60)\text{ kN}\cdot\text{m}$$

PROBLEM 5.10 *(Continued)*



$$+\uparrow \Sigma F_y = 0: \quad V + 48 = 0$$

$$V = -48 \text{ kN}$$

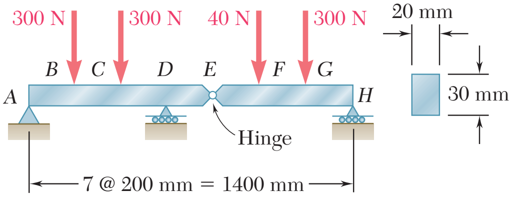
$$+\circlearrowleft \Sigma M_J = 0: \quad -M + 48(5 - x)$$

$$M = 240 - 48x$$

From the diagrams

(*a*) $|V|_{\max} = 72.0 \text{ kN}$ ◀

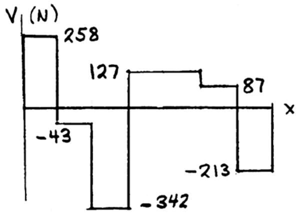
(*b*) $|M|_{\max} = 96.0 \text{ kN} \cdot \text{m}$ ◀

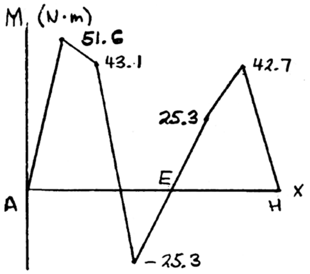


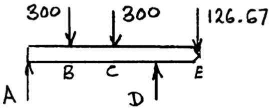
PROBLEM 5.23

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION







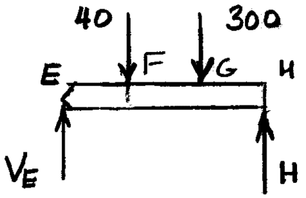
Free body *EFGH*. Note that $M_E = 0$ due to hinge.

$$+\circlearrowleft \Sigma M_E = 0: \quad 0.6\,H - (0.2)(40) - (0.40)(300) = 0$$

$$H = 213.33 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad V_E - 40 - 300 + 213.33 = 0$$

$$V_E = 126.67 \text{ N}$$



Shear:

E to F: $V = 126.67 \text{ N} \cdot \text{m}$

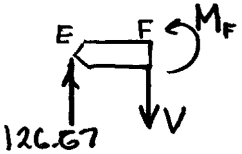
F to G: $V = 86.67 \text{ N} \cdot \text{m}$

G to H: $V = -213.33 \text{ N} \cdot \text{m}$

Bending moment at F:

$$+\circlearrowleft \Sigma M_F = 0: \quad M_F - (0.2)(126.67) = 0$$

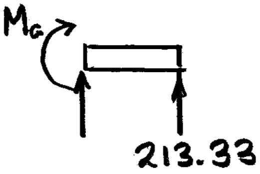
$$M_F = 25.33 \text{ N} \cdot \text{m}$$



Bending moment at G:

$$+\circlearrowleft \Sigma M_G = 0: \quad -M_G + (0.2)(213.33) = 0$$

$$M_G = 42.67 \text{ N} \cdot \text{m}$$



Free body *ABCDE*.

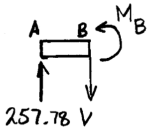
$$+\circlearrowleft \Sigma M_B = 0: \quad 0.6A + (0.4)(300) + (0.2)(300)$$
$$-(0.2)(126.63) = 0$$

$$A = 257.78 \text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6D = 0$$

$$D = 468.89 \text{ N}$$

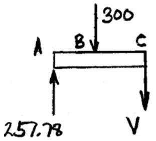
PROBLEM 5.23 *(Continued)*



Bending moment at *B*.

$$+\circlearrowleft \Sigma M_B = 0: \ -(0.2)(257.78) + M_B = 0$$

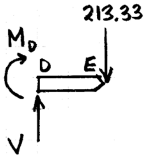
$$M_B = 51.56 \text{ N} \cdot \text{m}$$



Bending moment at *C*.

$$+\circlearrowleft \Sigma M_C = 0: \ -(0.4)(257.78) + (0.2)(300) + M_C = 0$$

$$M_C = 43.11 \text{ N} \cdot \text{m}$$



Bending moment at *D*.

$$+\circlearrowleft \Sigma M_D = 0: \ -M_D - (0.2)(213.33) = 0$$

$$M_D = -25.33 \text{ N} \cdot \text{m}$$

$$\max|M| = 51.56 \text{ N} \cdot \text{m}$$ ◀

$$S = \frac{1}{6}bh^2 = \frac{1}{6}(20)(30)^2$$

$$= 3 \times 10^3 \text{ mm}^3 = 3 \times 10^{-6} \text{ m}^3$$

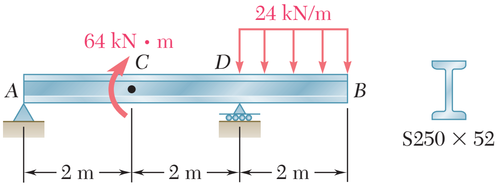
Normal stress:

$$\sigma = \frac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6 \text{ Pa}$$

$$\sigma = 17.19 \text{ MPa}$$ ◀

$$|V|_{\max} = 342 \text{ N}$$ ◀

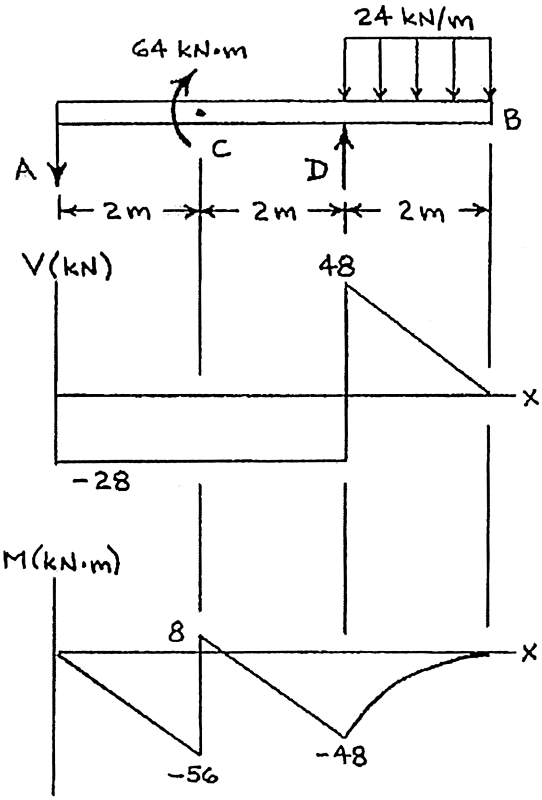
$$|M|_{\max} = 516 \text{ N} \cdot \text{m}$$ ◀



PROBLEM 5.24

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

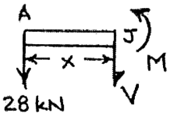


Reactions:

$+\circlearrowleft \Sigma M_D = 0$: $4A - 64 - (24)(2)(1) = 0$ $\quad \mathbf{A} = 28\text{ kN}\downarrow$

$+\uparrow \Sigma F_y = 0$: $-28 + D - (24)(2) = 0$ $\quad \mathbf{D} = 76\text{ kN}\uparrow$

A to *C*: $\quad 0 < x < 2\text{m}$



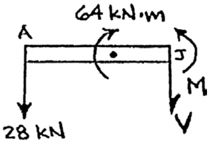
$+\uparrow \Sigma F_y = 0$: $\quad -V - 28 = 0$

$$V = -28\text{ kN}$$

$+\circlearrowleft \Sigma M_J = 0$: $\quad M + 28x = 0$

$$M = (-28x)\text{ kN}\cdot\text{m}$$

C to *D*: $\quad 2\text{m} < x < 4\text{m}$



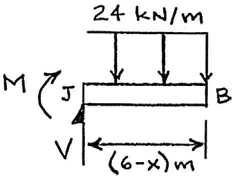
$+\uparrow \Sigma F_y = 0$: $\quad -V - 28 = 0$

$$V = -28\text{ kN}$$

$+\circlearrowleft \Sigma M_J = 0$: $\quad M + 28x - 64 = 0$

$$M = (-28x + 64)\text{ kN}\cdot\text{m}$$

D to *B*: $\quad 4\text{m} < x < 6\text{m}$



$+\uparrow \Sigma F_y = 0$:

$$V - 24(6 - x) = 0$$

$$V = (-24x + 144)\text{ kN}$$

$+\circlearrowleft \Sigma M_J = 0$:

$$-M - 24(6 - x)\left(\frac{6 - x}{2}\right) = 0$$

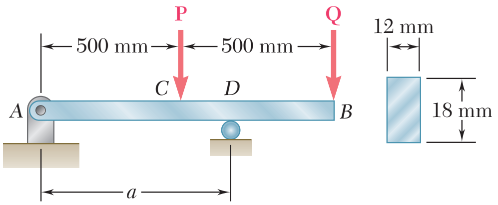
$$M = -12(6 - x)^2\text{ kN}\cdot\text{m}$$

$\max|M| = 56\text{ kN}\cdot\text{m} = 56 \times 10^3\text{ N}\cdot\text{m}$

For S250 × 52 section, $\quad S = 482 \times 10^3\text{ mm}^3$

Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{56 \times 10^3\text{N}\cdot\text{m}}{482 \times 10^{-6}\text{m}^3} = 116.2 \times 10^6\text{ Pa}$

$\sigma = 116.2\text{ MPa}$ ◀

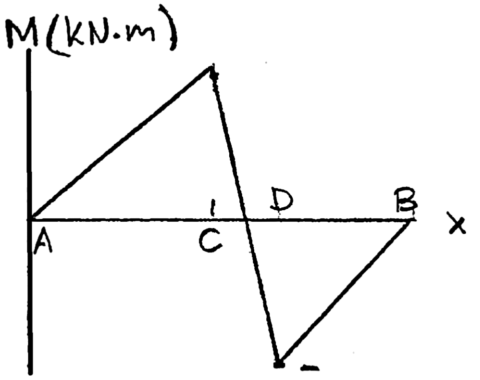


PROBLEM 5.29

Knowing that $P = Q = 480$ N, determine (*a*) the distance a for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

$$P = 480 \text{ N} \qquad Q = 480 \text{ N}$$



Reaction at A:

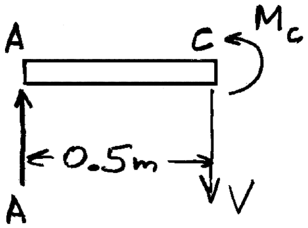
$$+\circlearrowleft \Sigma M_D = 0: \quad -Aa + 480(a - 0.5) - 480(1 - a) = 0$$

$$A = \left(960 - \frac{720}{a}\right) \text{ N}$$

Bending moment at C:

$$+\circlearrowleft \Sigma M_C = 0: \quad -0.5A + M_C = 0$$

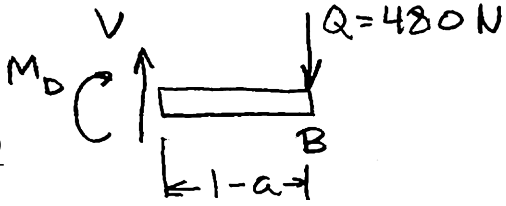
$$M_C = 0.5A = \left(480 - \frac{360}{a}\right) \text{ N} \cdot \text{m}$$



Bending moment at D:

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - 480(1 - a) = 0$$

$$M_D = -480(1 - a) \text{ N} \cdot \text{m}$$



(*a*) Equate: $-M_D = M_C \qquad 480(1 - a) = 480 - \dfrac{360}{a}$

$$a = 0.86603 \text{ m}$$

$a = 866$ mm ◀

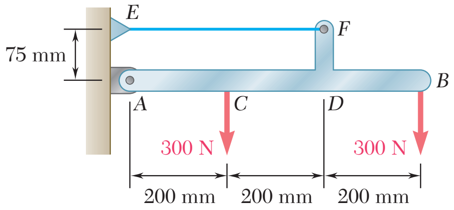
$$A = 128.62 \text{ N} \qquad M_C = 64.31 \text{ N} \cdot \text{m} \qquad M_D = -64.31 \text{ N} \cdot \text{m}$$

(*b*) For rectangular section, $S = \dfrac{1}{6}bh^2$

$$S = \frac{1}{6}(12)(13)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{m}^3$$

$$\sigma_{\text{max}} = \frac{|M|_{\text{max}}}{S} = \frac{64.31}{6.48 \times 10^{-9}} = 99.2 \times 10^6 \text{ Pa}$$

$\sigma_{\text{max}} = 99.2$ MPa ◀



PROBLEM 5.45

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

$$+\circlearrowleft \Sigma M_A = 0:$$

$$0.075\,F_{EF} - (0.2)(300) - (0.6)(300) = 0$$

$$F_{EF} = 3.2 \times 10^3 \text{ N}$$

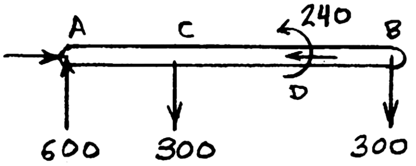
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - F_{EF} = 0 \quad A_x = 3.2 \times 10^3 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 300 - 300 = 0$$

$$A_y = 600 \text{ N}$$

Couple at *D*: $M_D = (0.075)(3.2 \times 10^3)$

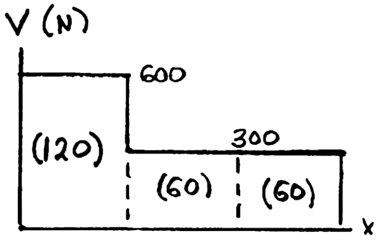
$= 240 \text{ N} \cdot \text{m}$



Shear:

A to *C*: $V = 600 \text{ N}$

C to *B*: $V = 600 - 300 = 300 \text{ N}$

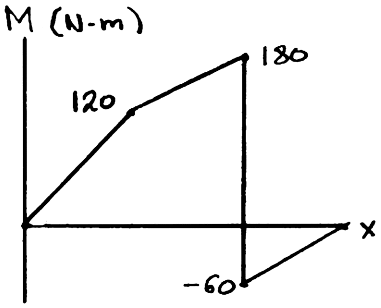


Areas under shear diagram:

A to *C*: $\int V\,dx = (0.2)(600) = 120 \text{ N} \cdot \text{m}$

C to *D*: $\int V\,dx = (0.2)(300) = 60 \text{ N} \cdot \text{m}$

D to *B*: $\int V\,dx = (0.2)(300) = 60 \text{ N} \cdot \text{m}$



Bending moments:

$M_A = 0$

$M_C = 0 + 120 = 120 \text{ N} \cdot \text{m}$

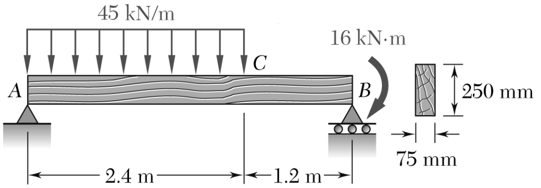
$M_{D^-} = 120 + 60 = 180 \text{ N} \cdot \text{m}$

$M_{D^+} = 180 - 240 = -60 \text{ N} \cdot \text{m}$

$M_B = -60 + 60 = 0$

Maximum $|V| = 600 \text{ N}$ ◀

Maximum $|M| = 180.0 \text{ N} \cdot \text{m}$ ◀



PROBLEM 5.57

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0$$

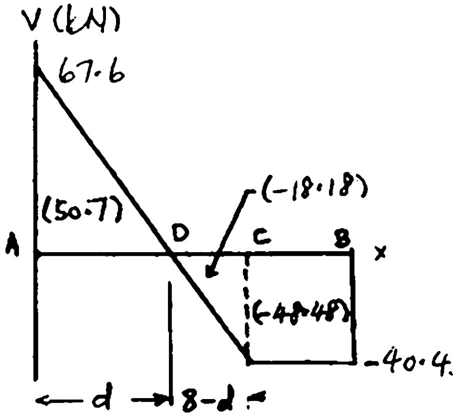
$$-3.6A + (45)(2.4)(2.4) - 16 = 0$$

$$A = 67.6 \text{ kN}$$

$$+\circlearrowleft \Sigma M_A = 0$$

$$-(45)(2.4)(1.2) + 3.6B - 16 = 0$$

$$B = 40.4 \text{ kN}$$



Shear: $V_A = 67.6 \text{ kN}$

$$V_C = 67.6 - (45)(2.4) = -40.4 \text{ kN}$$

C to B $\quad V = -40.4 \text{ kN}$

Locate point D where $V = 0$

$$\frac{d}{67.6} = \frac{2.4 - d}{40.4} \qquad 1.6d = 2.4$$

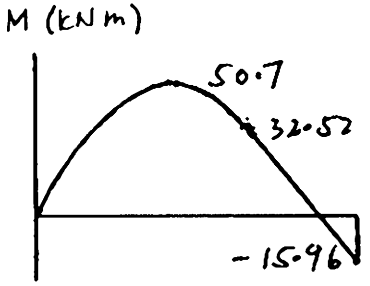
$$d = 1.5 \text{ m} \qquad 2.4 - d = 0.9 \text{ m}$$

Areas under shear diagram

A to D $\quad \int V dx = \left(\frac{1}{2}\right)(1.5)(67.6) = 50.7 \text{ kNm}$

D to C $\quad \int V dx = \left(\frac{1}{2}\right)(0.9)(-40.4) = -18.18 \text{ kNm}$

C to B $\quad \int V dx = -(1.2)(40.4) = -48.48 \text{ kNm}$



Bending moments: $M_A = 0$

$$M_D = 0 + 50.7 = 50.7 \text{ kNm}$$

$$M_C = 50.7 - 18.18 = 32.52 \text{ kNm}$$

$$M_B = 32.52 - 48.48 = -15.96 \text{ kNm}$$

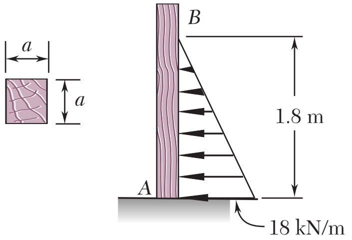
Maximum $|M| = 50.7$ kNm

For rectangular cross section $\quad S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(75)(250)^2 = 781250 \text{ mm}^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{50.7 \times 10^3}{781250 \times 10^{-9}} = 64.9 \text{ MPa}$ ◀

PROBLEM 5.67

5.67 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



SOLUTION

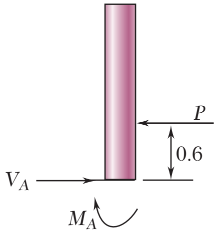
Equivalent concentrated load

$$P = \left(\frac{1}{2}\right)(1.8)(18) = 16.2 \text{ kN}$$

Bending moment at A

$$M_A = (0.6)(16.2) = 9.72 \text{ kN}\cdot\text{m}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{9.72 \times 10^3}{12 \times 10^6} = 810 \times 10^{-6} \text{ m}^3$$

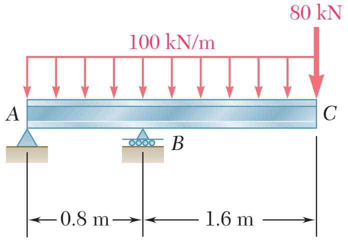


For a square section $S = \frac{1}{6} a^3$

$$a = \sqrt[3]{6S}$$

$$a_{\min} = \sqrt[3]{(6)(810 \times 10^{-6})} = 0.169 \text{ m}$$

$$= 169 \text{ mm}$$ ◄



PROBLEM 5.77

Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.

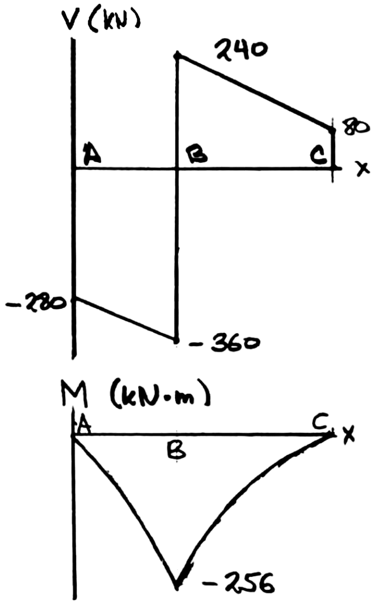
SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.8A - (0.4)(2.4)(100) - (1.6)(80) = 0$$

$$A = 280 \text{ kN} \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.8B - (1.2)(2.4)(100) - (2.4)(80) = 0$$

$$B = 600 \text{ kN} \uparrow$$



Shear: $V_A = -280$ kN

$$V_{B^-} = -280 - (0.8)(100) = -360 \text{ kN}$$

$$V_{B^+} = -360 + 600 = 240 \text{ kN}$$

$$V_C = 240 - (1.6)(100) = 80 \text{ kN}$$

Areas under shear diagram:

A to *B*: $\frac{1}{2}(0.8)(-280 - 360) = -256 \text{ kN} \cdot \text{m}$

B to *C*: $\frac{1}{2}(1.6)(240 + 80) = 256 \text{ kN} \cdot \text{m}$

Bending moments: $M_A = 0$

$$M_B = 0 - 256 = -256 \text{ kN} \cdot \text{m}$$

$$M_C = -256 + 256 = 0$$

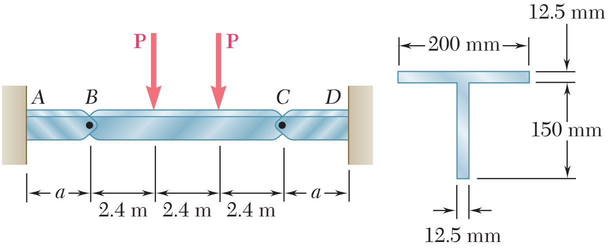
Maximum $|M| = 256 \text{ kN} \cdot \text{m} = 256 \times 10^3 \text{ N} \cdot \text{m}$

$$\sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{256 \times 10^3}{160 \times 10^6} = 1.6 \times 10^{-3} \text{ m}^3 = 1600 \times 10^3 \text{ mm}^3$$

Shape	S (10^3 mm^3)
S510×98.2	1950
S460×104	1685

Lightest S-section: S510×98.2 ◀

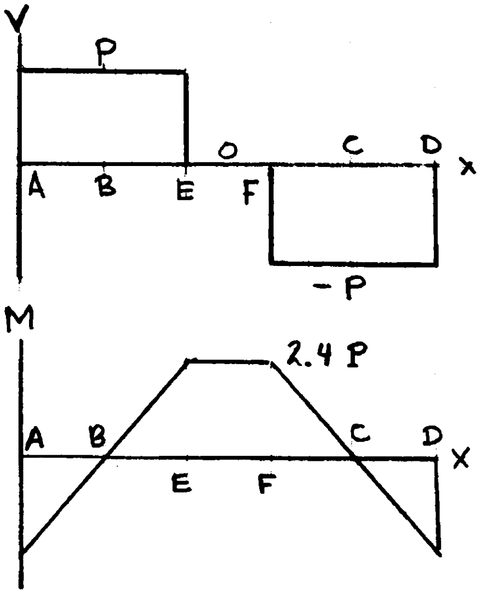


PROBLEM 5.90

Beams *AB*, *BC*, and *CD* have the cross section shown and are pin-connected at *B* and *C*. Knowing that the allowable normal stress is +110 MPa in tension and –150 MPa in compression, determine (*a*) the largest permissible value of **P** if beam *BC* is not to be overstressed, (*b*) the corresponding maximum distance *a* for which the cantilever beams *AB* and *CD* are not overstressed.

SOLUTION

$$M_B = M_C = 0$$
$$V_B = -V_C = P$$



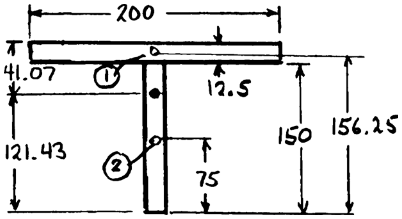
Area *B* to *E* of shear diagram: $2.4P$

$$M_E = 0 + 2.4P = 2.4P = M_F$$

Centroid and moment of inertia:

Part	$A\,(\text{mm}^2)$	$\bar{y}\,(\text{mm})$	$A\bar{y}\,(\text{mm}^3)$	$d\,(\text{mm})$	$Ad^2\,(\text{mm}^4)$	$\bar{I}\,(\text{mm}^4)$
①	2500	156.25	390,625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140,625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531,250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531{,}250}{4375} = 121.43 \text{ mm}$$
$$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621 \times 10^6 \text{ mm}^4$$



Location	$y\,(\text{mm})$	$I/y\,(10^3\text{ mm}^3)$
Top	41.07	258.6
Bottom	–121.43	–87.47

← also (10^{-6} m^3)

PROBLEM 5.90 *(Continued)*

Bending moment limits: $M = -\sigma I/y$

Tension at E and F: $-(110\times10^{6})(-87.47\times10^{-6}) = 9.622\times10^{3}\ \text{N}\cdot\text{m}$

Compression at E and F: $-(-150\times10^{6})(258.6\times10^{-6}) = 38.8\times10^{3}\ \text{N}\cdot\text{m}$

Tension at A and D: $-(110\times10^{6})(258.6\times10^{-6}) = -28.45\times10^{3}\ \text{N}\cdot\text{m}$

Compression at A and D: $-(-150\times10^{6})(-87.47\times10^{-6}) = -13.121\times10^{3}\ \text{N}\cdot\text{m}$

(*a*) Allowable load P: $2.4P = 9.622\times10^{3}$ $P = 4.01\times10^{3}\ \text{N}$ $P = 4.01\ \text{kN}$ ◀

Shear at A: $V_A = P$

Area A to B of shear diagram: $aV_A = aP$

Bending moment at A: $M_A = -aP = -4.01\times10^{3}a$

(*b*) Distance a: $-4.01\times10^{3}a = -13.121\times10^{3}$ $a = 3.27\ \text{m}$ ◀